

Unit 16

Exponentials and Logarithms

Day 1

The Exponential Function

Recall the following property of exponents:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a^m} \right)$$

example:

$$\begin{aligned} 32^{\frac{2}{5}} &= \sqrt[5]{32^2} \\ &= \left(\sqrt[5]{32} \right)^2 \end{aligned}$$

Additional Properties of Exponents:

For any real number $a > 0$, $a \neq 1$, and any real number x , the following statements are true.

(a) a^x is a unique real number.

(b) $a^b = a^c$ if and only if $b = c$.

(c) If $a > 1$ and $m < n$, then $a^m < a^n$.

(d) If $0 < a < 1$ and $m < n$, then $a^m > a^n$.

Solving Exponential Equations

1)

$$9^x = 3$$

$$(3^2)^x = 3$$

$$3^{2x} = 3^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

2)

$$\left(\frac{1}{5}\right)^x = 125$$

$$(5)^{-x} = 5^3$$

$$-x = 3$$

$$x = -3$$

$$3) \quad \frac{125}{64} = y^{-3}$$

$$\left(\frac{5}{4}\right)^3 = y^{-3}$$

$$\left(\frac{4}{5}\right)^{-3} = y^{-3}$$

$$\frac{4}{5} = y$$

$$4) \quad 16^x = 64$$

$$2^{4x} = 2^6$$

$$4x = 6$$

$$x = \frac{3}{2}$$

5) $25^p = 625$

$$5^{2p} = 5^4$$

$$2p = 4$$

$$p = 2$$

6) $100^{2+t} = 1000$

$$10^{4+2t} = 10^3$$

$$4+2t = 3$$

$$2t = -1$$

$$t = -\frac{1}{2}$$

7)

$$4^{k+2} = 32$$

$$2^{2k+4} = 2^5$$

$$2k+4=5$$

$$2k=1$$

$$k = \frac{1}{2}$$

8)

$$25^{1-t} = 5$$

$$5^{2-2t} = 5$$

$$2-2t=1$$

$$-2t = -1$$

$$t = \frac{1}{2}$$

9)

$$256^x = 64^{2-x}$$

$$2^{8x} = 2^{12-6x}$$

$$8x = 12 - 6x$$

$$14x = 12$$

$$x = \frac{6}{7}$$

10)

$$\left(\frac{1}{36}\right)^{2-r} = 216^{3r}$$

$$36^{r-2} = 216^{3r}$$

$$6^{2r-4} = 6^{9r}$$

$$2r-4 = 9r$$

$$-7r = 4$$

$$r = -\frac{4}{7}$$

12)

$$\left(\frac{1}{4}\right)^{x+1} = \left(\frac{1}{2}\right)^{x+1}$$

$$4^{-x-1} = 2^{-x-1}$$

$$2^{-2x-2} = 2^{-x-1}$$

$$-2x-2 = -x-1$$

$$-x = 1$$

$$x = -1 \quad \checkmark$$

13)

$$\left(\frac{3}{5}\right)^{-x} = \left(\frac{25}{9}\right)^{x-2}$$

$$\left(\frac{3}{5}\right)^{-x} = \left(\frac{5}{3}\right)^{2x-4}$$

$$\left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{2x-4}$$

$$x = 2x - 4$$

$$4 = x$$

Homework

Unit 16 Day 1

p. 370-371: 1-8, 49-62